Chapter 10 - Lecture 3
Analysis of Paired Data

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Introduction

Let $X_1, \ldots, X_m$ and let $Y_1, \ldots, Y_n$.

Until now we have seen how to handle cases when these two samples are independent. Now, what happens if the two samples are paired/matched?

Definition

Paired data usually refer to then when two measurements are taken on the same subject or one measurement is taken on the paired subjects.

- On subject i, one treatment for usage of drug A and another treatment for usage of drug B after wash-out period.
- Salary comparison between husband and wife.
- Case-control study (paired for research purpose).

Analysis for paired data

What's different for the paired data?

- First: Since the two samples are paired, that means there should be the same number of observations in each sample.
- Second: When the two samples are dependent, we no longer have $V(\bar{X} - \bar{Y}) = V(\bar{X}) + V(\bar{Y})$

Think: Can we construct the independent data?

Mathematical setting:

For the two samples $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$,

1. $m = n$;
2. The pairs $(X_1, Y_1), \ldots, (X_n, Y_n)$ are i.i.d.

Remember what we are interested in, is to get information about $\mu_1 - \mu_2$ from $\bar{X} - \bar{Y}$ (and sample variance), so we can use the information by taking difference:

1. $m = n$;
2. Let $D_1 = X_1 - Y_1, \ldots, D_n = X_n - Y_n$, then $D_1, \ldots, D_n$ are i.i.d.

Now, what we have is :

- $D_1, \ldots, D_n$ are i.i.d.
- $E(D_1) = \mu_D = \mu_1 - \mu_2$; $V(D_1) = \sigma_D^2$.

This becomes one sample case!

- C.I. about $\mu_1 - \mu_2$ is C.I. about μ_D
- Testing $\mu_1 = \mu_2$ is testing $\mu_D = 0$.

Everything is parallel to Chapters 8 and 9. z C.I., z-test, t C.I., t-test...

Here we only discuss the t C.I. (t test) named as **the paired t C.I.** (the **paired t test**). Other tests can be derived similarly.

So from now on in this section we assume $D_1, \ldots, D_n$ i.i.d $N(\mu_D, \sigma_D^2)$ with σ_D^2 unknown.

Confidence Interval

Let $\bar{D}$ and S_D be the sample mean and sample sd of $D_1, \ldots, D_n$. Then we have:

$$T = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} \sim t_{n-1}$$

paired t CI for μ_D is $\bar{d} \pm t_{\alpha/2,n-1} \cdot s_D/\sqrt{n}$. Similarly we can get one-sided CI's.

Hypothesis Testing

The paired t test

$H_0 : \mu_D = \Delta_0$

$$t = \frac{\bar{d} - \Delta_0}{s_D/\sqrt{n}}$$

- If $H_1 : \mu_D > \Delta_0$, we reject H_0 if $t \geq t_{\alpha,n-1}$;
- If $H_1 : \mu_D < \Delta_0$, we reject H_0 if $t \leq -t_{\alpha,n-1}$;
- If $H_1 : \mu_D \neq \Delta_0$, we reject H_0 if $|t| \geq t_{\alpha/2,n-1}$;

We can get the p values similarly as in Chapter 9.

Example 10.9

Musculoskeletal neck-and-shoulder disorders are all too common among office staff who perform repetitive tasks using visual display units. The article "Upper- Arm Elevation During Office Work"(Ergonomics, 1996: 12211230) reported on a study to determine whether more varied work conditions would have any impact on arm movement.

The accompanying data was obtained from a sample of $n = 16$ subjects. Each observation is the amount of time, expressed as a proportion of total time observed, during which arm elevation was below 30. The two measurements from each subject were obtained 18 months apart. During this period, work conditions were changed, and subjects were allowed to engage in a wider variety of work tasks. **Does the data suggest that true average time during which elevation is below 30 differs after the change from what it was before the change?** This particular angle is important because in Sweden, where the research was conducted, workers's compensation regulations assert that arm elevation less than 30 is not harmful.

Example 10.9 (cont.)

Subject	1	2	3	4	5	6	7	8
Before	81	87	86	82	90	86	96	73
After	78	91	78	78	84	67	92	70
Difference	3	−4	8	4	6	19	4	3
Subject	9	10	11	12	13	14	15	16
Before	74	75	72	80	66	72	56	82
After	58	62	70	58	66	60	65	73
Difference	16	13	2	22	0	12	−9	9

$$\bar{d} = 6.75, s_D = 8.234$$

Example 10.9 (cont.)

Check normality.

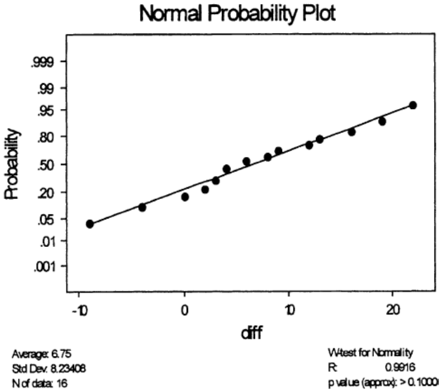

Figure 10.6 A normal probability plot from MINITAB of the differences in Example 10.9

Paired Versus Unpaired Experiments

Not an easy question!

1. Guidelines: If we have a choice between two t tests that are both valid (and carried out at the same level of significance a), we should prefer the test that has the larger number of degrees of freedom.
2. If there is great heterogeneity between experimental units and a large correlation within experimental units (large positive r), then the loss in degrees of freedom will be compensated for by the increased precision associated with pairing, so a paired experiment is preferable to an independent- samples experiment.
3. If the experimental units are relatively homogeneous and the correlation within pairs is not large, the gain in precision due to pairing will be outweighed by the decrease in degrees of freedom, so an independent samples experiment should be used.