Chapter 8 - Lecture 3
Intervals Based on a Normal Populations Distribution

Yuan Huang

February 18, 2013

1. Normal population with known variance

2. Normal population with unknown variance
   - Large Samples
   - Small Samples

Review

- Let a random sample $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$ where σ^2 is known. We are interested in constructing a $(1-\alpha)$ Confidence Interval for μ.

- We have seen this last lecture:

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

Interval estimation for Normal with unknown variance

- Let a random sample $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$ where σ^2 is unknown. We are interested in constructing a $(1 - \alpha)$ Confidence Interval for μ.
- How should we do this? There are two different cases:
  - Case 1: large sample: $n > 30$
  - Case 2: small sample: $n < 30$

Sample size greater than 30

- What do you think the best thing to do if sample size is greater then 30?

Sample size greater than 30

- What do you think the best thing to do if sample size is greater then 30?

For large sample size, we can apply the conclusion for general distribution with unknown variance, and get the approximate CI.

$$(\bar{x} - z_{\alpha/2} s/\sqrt{n}, \quad \bar{x} + z_{\alpha/2} s/\sqrt{n})$$

Sample size less than or equal to 30

- What do you think is the best thing to do if sample size is smaller than 30?

Sample size less than or equal to 30

- What do you think is the best thing to do if sample size is smaller than 30?

Proposition

If $X_1, \dots, X_n$ are i.i.d with $X_1 \sim N(\mu, \sigma^2)$, then

$$\frac{\sqrt{n}(\bar{X} - \mu)}{S} \sim t_{n-1}.$$

Normal with unknown variance under small sample size

Under $X_1, \ldots, X_n$ i.i.d $\sim N(\mu, \sigma^2)$, σ^2 unknown. $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$;

$\rightarrow P(-t_{\alpha/2,n-1} < \frac{\bar{X}-\mu}{S/\sqrt{n}} < t_{\alpha/2,n-1}) = 1-\alpha$

$\rightarrow P(\bar{X} - t_{\alpha/2,n-1}S/\sqrt{n} < \mu < \bar{X} + t_{\alpha/2,n-1}S/\sqrt{n}) = 1-\alpha$

Proposition

Exact $1-\alpha$ CI for μ for Normal with unknown variance under small sample size is

$$(\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \quad \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}})$$

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Example 8.11

There are alcohol percentages for a sample of 16 beers:
4.68, 4.13, 4.80, 4.63, 5.08, 5.79, 6.29, 6.79,
4.93, 4.25, 5.70, 4.74, 5.88, 6.77, 6.04, 4.95
Assume the percentage is normally distributed. Construct the 95% for the mean percentage.

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- $n = 16 < 30$
- $\bar{X} = \sum_{i=1}^{n} \to \bar{x} = 5.34$
- $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2 \to s = 0.8483$
- $t_{\alpha/2,n-1} = t_{0.05/2,16-1} = t_{0.025,15} = 2.131$
- 95% CI is $(\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \quad \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}) =$ $(5.34 - 2.131\frac{0.8483}{\sqrt{16}}, \quad 5.34 + 2.131\frac{0.8483}{\sqrt{16}}) = (4.89, 5.79)$