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Ch9.2 Tests about population Mean

In this lecture, we will learn three different tests about the population mean [1] :

- Case I: Normal population with known σ
- Case II: population with unknown σ and sample size $n > 40$
- Case III: population with unknown σ and sample size $n \leq 40$

The goal of this lecture

- Choose the correct tests about mean for different situations.
- Understand the one-sided and two-sided tests.
- Be able to work out the examples with the five steps testing procedures.
- Understand further calculation of type II error for Case I and II.

[1] Materials are from *Modern Mathematical Statistics with Applications, 2rd*,lec 9.2

Ch9.2 Tests about population Mean

0.1 Case I: Normal population with known σ

Null hypothesis: H_0: $\mu = \mu_0$

Test statistic value: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Alternative Hypothesis	Rejection Region for Level α Test	Type II Error Probability $\beta(\mu')$ for a Level α Test
H_a: $\mu > \mu_0$	$z \geq z_\alpha$ (upper-tailed test)	$\Phi\left(z_\alpha + \dfrac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$
H_a: $\mu < \mu_0$	$z \leq -z_\alpha$ (lower-tailed test)	$1 - \Phi\left(-z_\alpha + \dfrac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$
H_a: $\mu \neq \mu_0$	$z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$ (two-tailed test)	$\Phi\left(z_{\alpha/2} + \dfrac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) - \Phi\left(-z_{\alpha/2} + \dfrac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$

Illustration of type I and type II error.

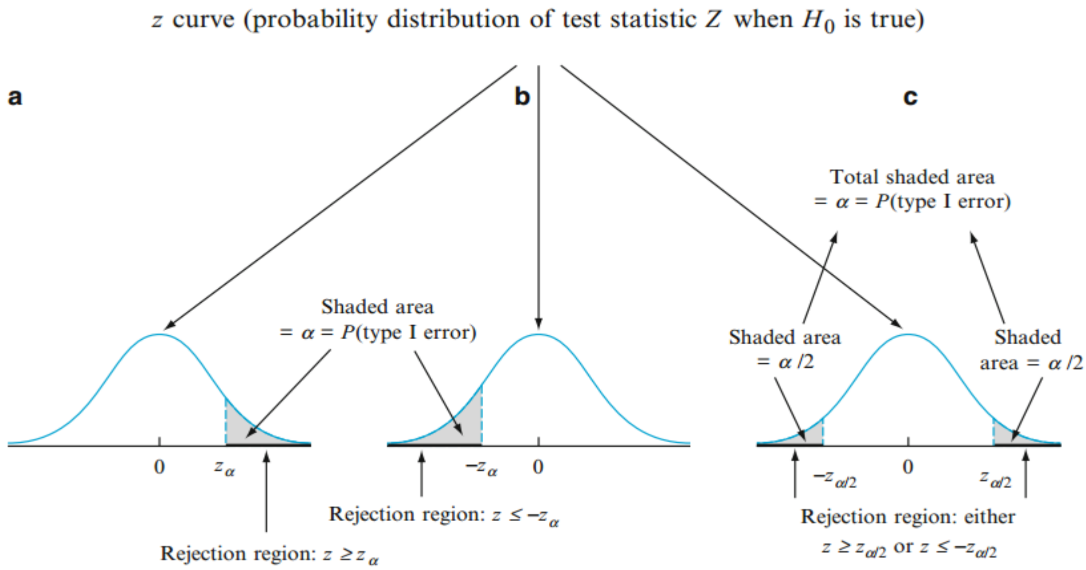


Figure 9.2 Rejection regions for z tests: (a) upper-tailed test; (b) lower-tailed test; (c) two-tailed test

Example 9.6 A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is 130°. A sample of $n = 9$ systems, when tested, yields a sample average activation temperature of 131.08°F. If the distribution of activation times is normal with standard deviation 1.5°F, does the data contradict the manufacturer's claim at significance level $\alpha = .01$?

1. Parameter of interest: μ = true average activation temperature.
2. Null hypothesis: H_0: $\mu = 130$ (null value $= \mu_0 = 130$).
3. Alternative hypothesis: H_a: $\mu \neq 130$ (a departure from the claimed value in *either* direction is of concern).
4. Test statistic value:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 130}{1.5/\sqrt{n}}$$

5. Rejection region: The form of H_a implies use of a two-tailed test with rejection region *either* $z \geq z_{.005}$ *or* $z \leq -z_{.005}$. From Section 4.3 or Appendix Table A.3, $z_{.005} = 2.58$, so we reject H_0 if either $z \geq 2.58$ or $z \leq -2.58$.
6. Substituting $n = 9$ and $\bar{x} = 131.08$,

$$z = \frac{131.08 - 130}{1.5/\sqrt{9}} = \frac{1.08}{.5} = 2.16$$

   That is, the observed sample mean is a bit more than 2 standard deviations above what would have been expected were H_0 true.
7. The computed value $z = 2.16$ does not fall in the rejection region ($-2.58 < 2.16 < 2.58$), so H_0 cannot be rejected at significance level .01. The data does not give strong support to the claim that the true average differs from the design value of 130. ■

Example 9.7 Let μ denote the true average tread life of a type of tire. Consider testing H_0: $\mu = 30{,}000$ versus H_a: $\mu > 30{,}000$ based on a sample of size $n = 16$ from a normal population distribution with $\sigma = 1500$. A test with $\alpha = .01$ requires $z_\alpha = z_{.01} = 2.33$. The probability of making a type II error when $\mu = 31{,}000$ is

$$\beta(31{,}000) = \Phi\left(2.33 + \frac{30{,}000 - 31{,}000}{1500/\sqrt{16}}\right) = \Phi(-.34) = .3669$$

■

0.2 Case II: population with unknown σ and sample size $n > 40$

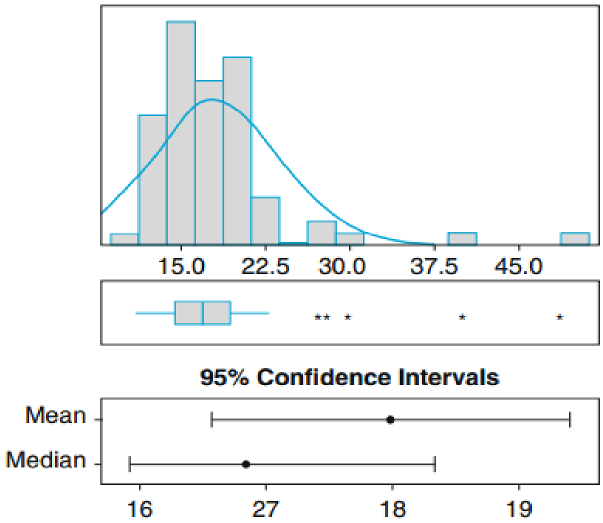
Null hypothesis: H_0: $\mu = \mu_0$

Test statistic value: $z = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$

Alternative Hypothesis	Rejection Region for Level α Test	Type II Error Probability $\beta(\mu')$ for a Level α Test
H_a: $\mu > \mu_0$	$z \geq z_\alpha$ (upper-tailed test)	$\Phi\left(z_\alpha + \dfrac{\mu_0 - \mu'}{s/\sqrt{n}}\right)$
H_a: $\mu < \mu_0$	$z \leq -z_\alpha$ (lower-tailed test)	$1 - \Phi\left(-z_\alpha + \dfrac{\mu_0 - \mu'}{s/\sqrt{n}}\right)$
H_a: $\mu \neq \mu_0$	$z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$ (two-tailed test)	$\Phi\left(z_{\alpha/2} + \dfrac{\mu_0 - \mu'}{s/\sqrt{n}}\right) - \Phi\left(-z_{\alpha/2} + \dfrac{\mu_0 - \mu'}{s/\sqrt{n}}\right)$

Example 9.8 A sample of bills for meals was obtained at a restaurant (by Erich Brandt). For each of 70 bills the tip was found as a percentage of the raw bill (before taxes). Does it appear that the population mean tip percentage for this restaurant exceeds the standard 15%? Here are the 70 tip percentages:

14.21   20.24   20.10   . . . . .   18.54   27.88   13.81



Anderson-Darting Normality Test	
A-Squared	4.17
P-Value <	0.005
Mean	17.986
StDev	5.937
Variance	35.247
Skewness	2.9391
Kurtosis	12.0154
N	70
Minimum	10.940
1st Quartile	14.540
Median	16.840
3st Quartile	19.358
Maximum	48.770
95% Confidence Interval for Mean	
16.571	19.402
95% Confidence Interval for Median	
15.913	18.402
95% Confidence Interval for StDev	
5.090	7.124

Figure 9.3 MINITAB descriptive summary for the tip data of Example 9.8

1. μ = true average tip percentage
2. H_0: $\mu = 15$
3. H_a: $\mu > 15$
4. $z = \dfrac{\bar{x} - 15}{s/\sqrt{n}}$
5. Using a test with a significance level .05, H_0 will be rejected if $z \geq 1.645$ (an upper tailed test).
6. With $n = 70$, $\bar{x} = 17.99$, and $s = 5.937$,

$$z = \frac{17.99 - 15}{5.937/\sqrt{70}} = \frac{2.99}{.7096} = 4.21$$

7. Since $4.21 > 1.645$, H_0 is rejected. There is evidence that the population mean tip percentage exceeds 15%. ■

0.3 Case III: population with unknown σ and sample size $n \leq 40$

Null hypothesis: H_0: $\mu = \mu_0$

Test statistic value: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

Alternative Hypothesis	**Rejection Region for Level α Test**
H_a: $\mu > \mu_0$	$t \geq t_{\alpha,n-1}$ (upper-tailed test)
H_a: $\mu < \mu_0$	$t \leq -t_{\alpha,n-1}$ (lower-tailed test)
H_a: $\mu \neq \mu_0$	$t \geq t_{\alpha/2,n-1}$ or $t \leq -t_{\alpha/2,n-1}$ (two-tailed test)

Example 9.9 A well-designed and safe workplace can contribute greatly to increased productivity. It is especially important that workers not be asked to perform tasks, such as lifting, that exceed their capabilities. The accompanying data on maximum weight of lift (MAWL, in kg) for a frequency of four lifts/min was reported in the article "The Effects of Speed, Frequency, and Load on Measured Hand Forces for a Floor-to-Knuckle Lifting Task" (*Ergonomics*, 1992: 833–843); subjects were randomly selected from the population of healthy males age 18–30. Assuming that MAWL is normally distributed, does the following data suggest that the population mean MAWL exceeds 25?

25.8	36.6	26.3	21.8	27.2

Let's carry out a test using a significance level of .05.

1. $\mu =$ population mean MAWL

2. H_0: $\mu = 25$

3. H_a: $\mu > 25$

4. $t = \dfrac{\bar{x} - 25}{s/\sqrt{n}}$

5. Reject H_0 if $t \geq t_{\alpha,\, n-1} = t_{.05,4} = 2.132$.

6. $\Sigma x_i = 137.7$ and $\Sigma x_i^2 = 3911.97$, from which $\bar{x} = 27.54$, $s = 5.47$, and

$$t = \frac{27.54 - 25}{5.47/\sqrt{5}} = \frac{2.54}{2.45} = 1.04$$

The accompanying MINITAB output from a request for a one-sample t test has the same calculated values (the P-value is discussed in Section 9.4).

```
Test of mu = 25.00 vs mu > 25.00
Variable   N   Mean    StDev   SE Mean   T      P-Value
mawl       5   27.54   5.47    2.45      1.04   0.18
```

7. Since 1.04 does not fall in the rejection region ($1.04 < 2.132$), H_0 cannot be rejected at significance level .05. It is still plausible that μ is (at most) 25. ■