Chapter 9 - Lecture 4
P-values

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Introduction

In the 5-step procedure, we are given a significant level α and calculate the rejection region. If the value of test statistic falls in the rejection region, we reject H_0; otherwise, fail to reject H_0.

Today, we introduce an alternative way of reaching a conclusion in hypothesis testing. In this approach, we will calculate a certain probability called *p-value*.

Definition

P-value is the probability, of obtaining a test statistic at least as contradictory to the null hypothesis as the one we have calculated from the available sample, assuming the null hypothesis is true.

- P-values are important for the following reason:
  - If p-value $\leq \alpha$ we reject H_0
  - If p-value $> \alpha$ we do not reject H_0
- So in the 5-step procedure of testing hypotheses, we can replace "Step 4: Determine the rejection/critical region C" with the following step:
  - Step 4: Calculate the p-value

Proposition

The P-value is the smallest significance level α at which the null hypothesis can be rejected. Because of this, the P-value is alternatively referred to as the **observed significance level** *for the data.*

Calculating p- values

How do we calculate p-values?

- p-values depend on the tests conducted. Hence, the calculation will be done most of the time by definition.

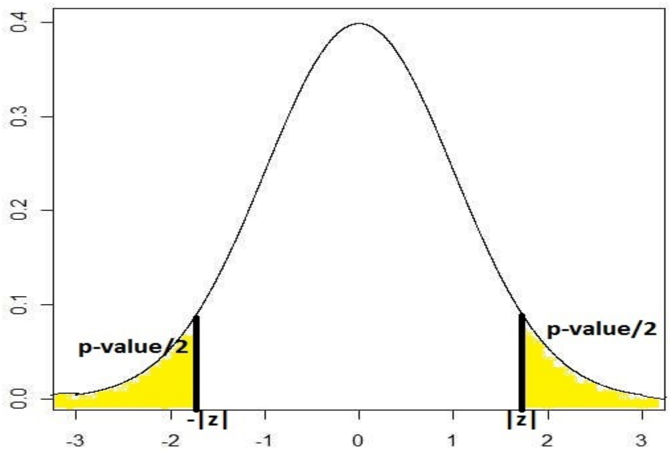
1. z test / t test;
2. upper-tailed / lower-tailed / two-tailed;

Suppose the test statistic is Z (or T), and the corresponding value in the sample is z (or t). Then the p-value is

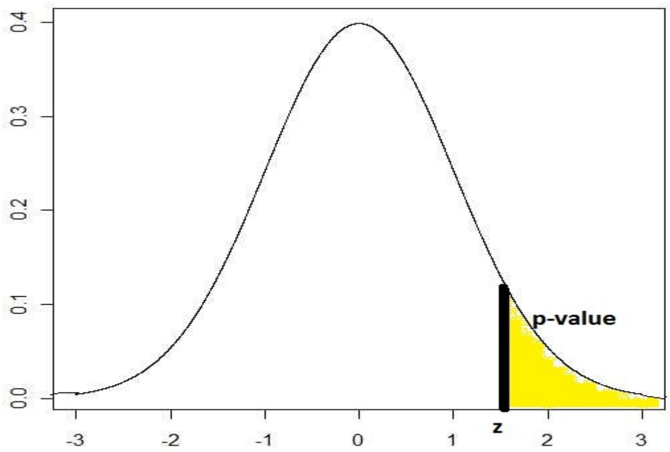
1. Two-sided test $H_1 : \mu \neq \mu_0$: $P(|Z| > z)$, $P(|T| > t)$;
2. Upper-tailed test $H_1 : \mu > \mu_0$: $P(Z > z)$, $P(T > t)$;
3. Lower-tailed test $H_1 : \mu < \mu_0$: $P(Z < z)$, $P(T < t)$.

Next 3 slides will use the z test as an example to illustrate the p-value. (z test has z curve and t test will have t_{n-1} curve.)

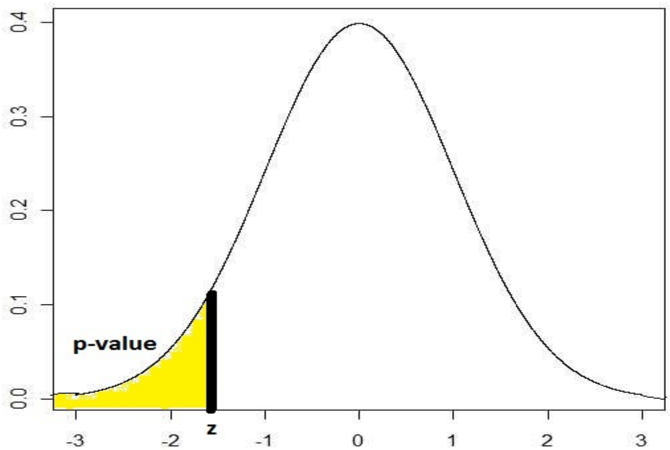
Two-sided test $H_1 : \mu \neq \mu_0$



Upper-tailed test $H_1 : \mu > \mu_0$



Lower-tailed test $H_1 : \mu < \mu_0$



Example 9.17

Example 9.17: The target thickness for silicon wafers used in a type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean thickness of 246.18 μm and a sample standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value?

Example 9.17 (cont.)

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step5: Based on the data, plug in $\bar{x} = 246.18$, $s = 3.60$
$\rightarrow z = 2.32 \rightarrow P - value = 2[1 - \Phi(2.32)] = 0.0204$.

- If $\alpha = 0.01$, fail to reject H_0.
- If $\alpha = 0.05$, reject H_0.