Math/Stat319 Formulas for chapter 6

Calculate mean and variance:

- $E(\sum_{i=1}^{n} a_i X_i) = a_1 E(X_1) + a_2 E(X_2) + \ldots + a_n E(X_n)$.
  1. $E(\bar{X}) = \mu$;
- $V(a_1 X_1 + \ldots + a_n X_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j Cov(X_i, X_j)$.
  1. $Cov(X_i, X_i) = V(X_i)$;
  2. If $X_1, \ldots, X_n$ are mutually independent are independent, $V(\sum_{i=1}^{n} a_i X_i) = \sum_{i=1}^{n} a_i^2 V(X_i)$.
  3. If $X_1, \ldots, X_n$ is random sample, $V(\bar{X}) = \frac{\sigma^2}{n}$
- When $X_1, \ldots, X_n$ are independent and normally distributed, suppose $X_i \sim N(\mu_i, \sigma_i^2)$, then for any linear combination $Y = a_1 X_1 + \ldots + a_n X_n = \sum_{i=1}^{n} a_i X_i$,

$$Y \sim N(\sum_{i=1}^{n} a_i \mu_i, \sum_{i=1}^{n} a_i^2 \sigma_i^2).$$

  1. A random sample $X_1, X_2, ..., X_n$ from a normal distribution with mean μ and variance σ^2 $\left(X_i \sim N(\mu, \sigma^2)\right)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$

Tools:
Let $X_1, X_2, \ldots, X_n$ independent random variables with mgfs $M_{X_i}(t)$ and Y then for any linear combination $Y = a_1 X_1 + \ldots + a_n X_n = \sum_{i=1}^{n} a_i X_i$, then

$$M_Y(t) = M_{X_1}(a_1 t) \times M_{X_2}(a_2 t) \times \ldots \times M_{X_n}(a_n t)$$

LLN: If $X_1, \ldots, X_n$ is a random sample from a distribution with mean μ and variance σ^2, then as $n \to \infty$, $\bar{X}_n$ converges to μ :

- In mean square $E[(\bar{X} - \mu)^2] \to 0$
- In probability $P(|\bar{X} - \mu| \geq \epsilon) \to 0$

CLT:. If $X_1, \ldots, X_n$ is a random sample with mean μ and variance σ^2, then as $n \to \infty$, the limiting distribution of $\sqrt{n}(\bar{X}_n - \mu)/\sigma$ is standard normal, written as

$$\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} \to_d N(0, 1).$$

Distributions:

- χ^2_ν distribution.

  - If $Z \sim N(0,1)$, then $X = Z^2 \sim \chi^2_1$
  - If $Z_1, \ldots, Z_n$ are i.i.d and $Z_1 \sim N(0,1)$, then $X = \sum_{i=1}^n Z_i^2 \sim \chi^2_n$.

    1. If $X_1 \sim \chi^2_{\nu_1}$ and $X_2 \sim \chi^2_{v_2}$ and X_1, X_2 independent, then $X_1 + X_2 \sim \chi^2_{\nu_1+\nu_2}$
    2. If $X_3 = X_1 + X_2$, with $X_1 \sim \chi^2_{\nu_1}$, $X_3 \sim \chi^2_{\nu_3}$, $\nu_3 > \nu_1$ and X_1, X_2 independent, then $X_2 \sim \chi^2_{\nu_3-\nu_1}$

  - Let a random sample $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$, then

    1. $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$
    2. $\bar{X}$ and S^2 are independent
    3. $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

- t_ν distribution.

  If $Z \sim N(0,1)$, $X \sim \chi^2_v$ and X, Z are independent then

$$T = \frac{Z}{\sqrt{\frac{X}{v}}} \sim t_v$$

  1. If $X_1, \ldots, X_n$ are i.i.d with $X_1 \sim N(\mu, \sigma^2)$, then

$$\frac{\sqrt{n}(\bar{X} - \mu)}{S} \sim t_{n-1}.$$

- F_{df_1, df_2} distribution.

  If $X_1 \sim \chi^2_{v_1}$, $X_2 \sim \chi^2_{v_2}$ and X_1, X_2 are independent, then:

$$F = \frac{\frac{X_1}{v_1}}{\frac{X_2}{v_2}} \sim F_{v_1, v_2}$$